TYJ- 01 B MATHEMATICS SOLUTION 13 OCTOBER 2019

31. (c) Given lines are $2x-y+7=0$ and $2x-y+5=0$

Both the lines are on same side of origin.

Distnace between two parallel lines = $\dfrac{7-5}{\sqrt{2^2+1^2}}=\dfrac{2}{\sqrt{5}}$.

32. (b) $L \equiv 3x-4y-8=0$

$L_{(3,4)}=9-16-8<0$ and $L_{(2,-6)}=6+24-8>0$

Hence, the points lie on different side of the line.

33. (a) It is given that the lines $ax+2y+1=0$, $bx+3y+1=0$ and $cx+4y+1=0$ are concurrent, therefore

$$\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix}=0$$

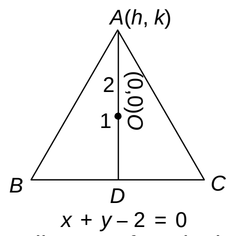
$\Rightarrow\ -a+2b-c=0 \Rightarrow 2b=a+c$

$\Rightarrow\ a,b,c$ are in A. P.

34. (b) $-y+3x+4=0$ and perpendicular is $\dfrac{y-3}{x-2}=\dfrac{-1}{3}$ or $3y+x-11=0$. Therefore foot is $x=\dfrac{-1}{10}, y=\dfrac{37}{10}$.

35. (c) Let the co-ordinate of vertex A be (h,k). Then AD is perpendicular to BC, therefore $OA \perp BC$

$\Rightarrow \dfrac{k-0}{h-0}\times\dfrac{-1}{1}=-1 \Rightarrow k=h$ .....(i)



Let the coordinates of D be (α,β). Then the co-ordinates of O are $\left(\dfrac{2\alpha+h}{2+1},\dfrac{2\beta+k}{2+1}\right)$. Therefore

$\dfrac{2\alpha+h}{3}=0$ and $\dfrac{2\beta+k}{3}=0 \Rightarrow \alpha=-\dfrac{h}{2}, \beta=\dfrac{-k}{2}$.

Since (α,β) lies on $x+y-2=0 \Rightarrow \alpha+\beta-2=0$

$\Rightarrow\ -h/2-k/2-2=0 \Rightarrow h+k+4=0$

$\Rightarrow\ 2h+4=0 \Rightarrow h=k=-2$, [from (i)]

Hence the coordinates of vertex A are $(-2,-2)$.

36. (a) Equation of the line passing through (3, 8) and perpendicular to $x+3y-7=0$ is $3x-y-1=0$. The intersection point of both the lines is (1, 2).

Now let the image of $A(3,8)$ be $A'(x_1,y_1)$, then point (1, 2) will be the mid point of AA'.

$\Rightarrow \dfrac{x_1+3}{2}=1 \Rightarrow x_1=-1$ and $\dfrac{y_1+8}{2}=2 \Rightarrow y_1=-4$.

Hence the image is (–1, –4).

37. (b) Let the co-ordinates of the third vertex be $(2a,\ t)$.

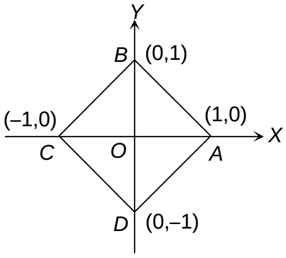
$AC=BC \Rightarrow t=\sqrt{4a^2+(a-t)^2} \Rightarrow t=\dfrac{5a}{2}$

So the coordinates of third vertex C are $\left(2a,\dfrac{5a}{2}\right)$

Therefore area of the triangle

$$= \pm\frac{1}{2}\begin{vmatrix} 2a & \frac{5a}{2} & 1 \\ 2a & 0 & 1 \\ 0 & a & 1 \end{vmatrix} = \begin{vmatrix} a & \frac{5a}{2} & 1 \\ 0 & -\frac{5a}{2} & 0 \\ 0 & a & 1 \end{vmatrix} = \frac{5a^2}{2} sq.\, units.$$

38. (a) Required locus of the point (x, y) is the curve $|x| + |y| = 1$. If the point lies in the first quadrant, then $x > 0, y > 0$ and so $|x| + |y| = 1 \Rightarrow x + y = 1$, which is straight line *AB*. If the point (x, y) lies in second quadrant then $x < 0$, $y > 0$ and so $|x| + |y| = 1 \Rightarrow -x + y = 1$



Similarly for third and fourth quadrant, the equations are $-x - y = 1$ and $x - y = 1$.
Hence the required locus is the curve consisting of the sides of the square *ABCD*.

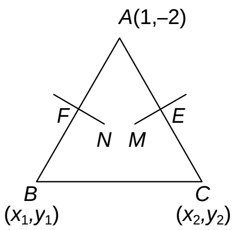
39. (c) According to question , $x_1 = \frac{2+4+x}{3} \Rightarrow x = 3x_1 - 6$

$y_1 = \frac{5-11+y}{3} \Rightarrow y = 3y_1 + 6$

$\therefore\ 9(3x_1 - 6) + 7(3y_1 + 6) + 4 = 0$

Hence locus is $27x + 21y - 8 = 0$, which is parallel to $9x + 7y + 4 = 0$.

40. (d) Let the equation of perpendicular bisector *FN* of *AB* is $x - y + 5 = 0$ ......(i)



The middle point *F* of *AB* is $\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$ lies on line (i). Therefore $x_1 - y_1 = -13$ .....(ii)

Also *AB* is perpendicular to *FN*. So the product of their slopes is –1.

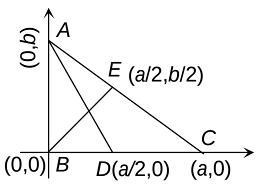
i.e. $\frac{y_1+2}{x_1-1} \times 1 = -1$ or $x_1 + y_1 = -1$ ......(iii)

On solving (ii) and (iii), we get $B(-7, 6)$.

Similarly $C\left(\frac{11}{5}, \frac{2}{5}\right)$.

Hence the equation of *BC* is $14x + 23y - 40 = 0$.

41. (c) From figure,



$\left(\frac{b/2}{a/2}\right)\left(\frac{b}{-a/2}\right) = -1 \Rightarrow a^2 = 2b^2 \Rightarrow a = \pm\sqrt{2}b$.

42. (c) Any line through (1, –10) is given by $y+10=m(x-1)$

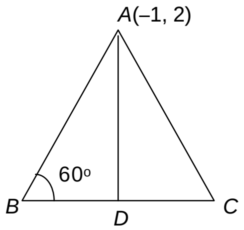
Since it makes equal angle say 'α' with the given lines $7x-y+3=0$ and $x+y-3=0$, therefore

$$\tan\alpha=\frac{m-7}{1+7m}=\frac{m-(-1)}{1+m(-1)}\Rightarrow m=\frac{1}{3}\text{ or }-3$$

Hence the two possible equations of third side are $3x+y+7=0$ and $x-3y-31=0$.

43. (a) $AD=\left|\dfrac{-2-2-1}{\sqrt{(2)^2+(-1)^2}}\right|=\left|\dfrac{-5}{\sqrt{5}}\right|=\sqrt{5}$

$$\because\ \tan 60^o=\frac{AD}{BD}\Rightarrow\sqrt{3}=\frac{\sqrt{5}}{BD}\Rightarrow BD=\sqrt{\frac{5}{3}}$$



$2x-y=1$

$$\therefore\ BC=2BD=2\sqrt{\frac{5}{3}}=\sqrt{\frac{20}{3}}.$$

44. (d) $m_1=-1/3$ and $m_2=3$. Hence lines $x+3y=4$ and $6x-2y=7$ are perpendicular to each other. Therefore the paralellogram is rhombus.

45. (b) Area of the right angled triangle is

$$=\frac{1}{2}\text{(Perpendicular)}\times\text{(base)}=\frac{1}{2}ab.$$